

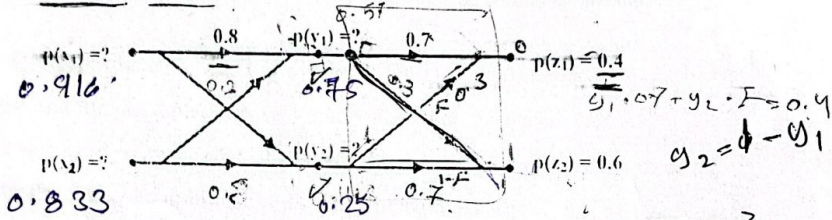
Q1. A discrete source transmits messages  $x_1, x_2, x_3$  with probabilities  $p(x_1) = 0.3, p(x_2) = 0.25, p(x_3) = 0.45$ . The source is connected to the channel whose conditional probability matrix is

$$P(Y/X) = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

A. Draw the channel schematic? And Obtain the joint probability matrix  $P(X, Y)$ ? (4 Marks)

B. Obtain the probabilities  $p(y_1), p(y_2)$  and  $p(y_3)$ ? (6 Marks)

Q2. Consider two binary symmetrical channels are connected in cascade as shown below:



A. Find the probabilities  $p(x_1), p(x_2), p(y_1)$  and  $p(y_2)$ ? (8 Marks)

Q3. (16 Marks) Consider a source with a six-symbol alphabet,  $x_1, x_2, x_3, x_4, x_5$ , and  $x_6$ , with probabilities  $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02$ , and  $P_6 = 0.04$ , respectively.

A. Find a Shannon-Fano code for this source. (8 Marks)

B. Compute the average code length of the code. (8 Marks)

Q4. (14 Marks) Consider the Systematic Linear Block Code with the following parity check matrix.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

A. If the code is a single error correction code, find the syndrome look-up table. (5 Marks)

B. For the received vectors  $r_1$  and  $r_2$ , recover the transmitted code-word and the original message. (6 Marks)

$r_1 = 1101011, r_2 = 0101101$ . (clearly show the recovery steps)

Q5. (15 Marks) Given the following trellis diagram of a convolutional code.

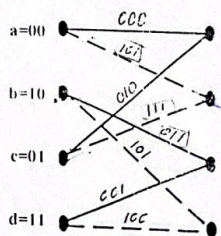
A. Encode the input sequence bits  $m = 10101$ . (1/0/0)

B. Decode the received vector  $r$  using Viterbi Decoding.

$R = 101, 111, 001, 000, 000.$

C. How many errors are there in the received vector  $R$

(identify the error bits) and show the transmitted vector  $U$



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### Information Theory and Coding (CM303)

Midterm Exam (40%)

07 June, 2015

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1. What is the maximum entropy  $H(s)$  for Binary Discrete Memoryless Source (BDMS)? Prove your answer? (10 points)

$$\frac{\text{dash}}{0.6}$$

Q2. A telegraph source having two symbols, dot and dash. The dash duration is 0.6 seconds; and the dot duration is two third of the dash duration. The probability of the dot occurring is twice that of the dash, and the time between symbols is 0.2 seconds. Calculate the information rate of the telegraph source? (10 points)

$$p(\text{dot}) = 2p(\text{dash})$$

Q3. A discrete memoryless source has an alphabet of seven symbols with probabilities for its output as described in following table:

Symbol	$S_0$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

1. Construct a Shannon-Fano code for the source and calculate the efficiency of coding? (10 points)
2. Construct a Huffman code for the source and calculate the efficiency of coding? And compare the results? (10 points)

Good luck!

# Information Theory and Coding (CM303)

Answer of Midterm Exam (40%)

07 June, 2015

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

## Q1. Answer:

Since it is a binary source, then the source has two symbols  $s_1$  and  $s_2$ . Let the probability of  $s_1$  is  $p(s_1)=a$  then the probability of  $s_2$  is  $p(s_2)=1-a$ . The entropy  $H(s)$  of this source:

$$H(s) = -a \log_2(a) - (1-a) \log_2(1-a)$$

Note that when  $a=0 \rightarrow H(s)=0$

$$a=1 \rightarrow H(s)=0$$

The maximum entropy can be found by the differentiation of  $H(s)$ :

$$\frac{dH(s)}{da} = \frac{d(-a \log_2(a) - (1-a) \log_2(1-a))}{da} = -[\log_2(a) + 1 - \log_2(1-a)] = -1 - \log_2(a) + \log_2(1-a)$$

$$\frac{dH(s)}{da} = -\log_2(a) + \log_2(1-a)$$

$$\frac{dH(s)}{da} = \log_2\left(\frac{1-a}{a}\right)$$

The maximum is found when  $\frac{dH(s)}{da} = 0$

$$\log_2\left(\frac{1-a}{a}\right) = 0 \text{ when } \frac{1-a}{a} = 1$$

$$\therefore a=0.5$$

Which means when  $a=0.5$   $H(s)$  is maximum

$$H(s) = -0.5 \log_2(0.5) - (1-0.5) \log_2(1-0.5) = 1 \text{ bit/symbol}$$



## Q2. Answer:

- Given that:
1. Dash duration: 0.6 sec.
  2. Dot duration:  $2/3 \times 0.6 = 0.4$  sec.
  3.  $P(\text{dot}) = 2 P(\text{dash})$ .
  4. Space between symbols is 0.2 sec.
- Information rate = ?

### 1. Probabilities of dots and dashes:

Let the probability of a dash be "P". Therefore the probability of a dot will be "2P". The total probability of transmitting dots and dashes is equal to 1.

$$\therefore P(\text{dot}) + P(\text{dash}) = 1$$

$$\therefore P + 2P = 1 \quad \text{P} = 1/3$$

$$\therefore \text{Probability of dash} = 1/3$$

$$\text{And Probability of dot} = 2/3$$

### 2. Average information H (X) per symbol:

$$\therefore H(X) = P(\text{dot}) \cdot \log_2 [1/P(\text{dot})] + P(\text{dash}) \cdot \log_2 [1/P(\text{dash})]$$

$$\therefore H(X) = (2/3) \log_2 [3/2] + (1/3) \log_2 [3] = 0.3899 + 0.5283 = 0.9182 \text{ bits/symbol.}$$

### 3. Symbol rate (Number of symbols/sec.):

The total average time per symbol can be calculated as follows:

$$\text{Average symbol time } T_s = [T_{\text{DOT}} \times P(\text{DOT})] + [T_{\text{DASH}} \times P(\text{DASH})] + T_{\text{space}}$$

$$\therefore T_s = [0.4 \times 2/3] + [0.6 \times 1/3] + 0.2 = 0.6667 \text{ sec/symbol.}$$

Hence the average rate of symbol transmission is given by:

$$R_s = 1/T_s = 1.5000 \text{ symbols/sec.}$$

### 4. Information rate (R<sub>I</sub>):

$$R_I = R_s \times H(s) = 1.5000 \times 0.9182 = 1.72 \text{ bits/sec.}$$

1.3773



### Q3. Answer:

1. Shannon-Fano code:

Symbols	Probability	Step 1	Step 2	Step 3	Step 4	Code word
$S_0$	0.25	0	0			00
$S_1$	0.25	0	1			01
$S_2$	0.125	1	0	0		100
$S_3$	0.125	1	0	1		101
$S_4$	0.125	1	1	0		110
$S_5$	0.0625	1	1	1	0	1110
$S_6$	0.0625	1	1	1	1	1111

Average code word length (L):

$$L = \sum_{k=0}^6 p_i \times n_i$$

$$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$$

$$= 2.6250 \text{ bits/message}$$

Entropy of the source (H):

$$H(s_i) = \sum_{k=0}^6 p_i \times \log_2(1/p_i)$$

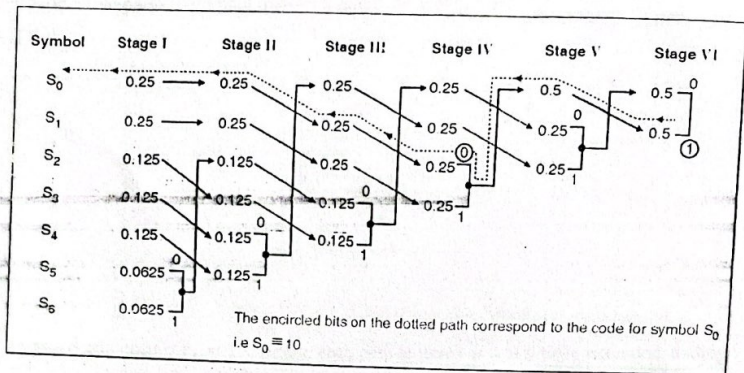
$$= 0.25 \log_2(1/0.25) + 0.25 \log_2(1/0.25) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) +$$

$$0.125 \log_2(1/0.125) + 0.0625 \log_2(1/0.0625) + 0.0625 \log_2(1/0.0625) = 2.6250 \text{ bits/symbols}$$

$$\text{Code efficiency } \eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$$

$$\therefore \eta = 100\%$$

## 2. Huffman Code:



Symbol	Probability	Codeword	Codeword length
$S_0$	0.25	10	2 bit
$S_1$	0.25	11	2 bit
$S_2$	0.125	001	3 bit
$S_3$	0.125	010	3 bit
$S_4$	0.125	011	3 bit
$S_5$	0.0625	0000	4 bit
$S_6$	0.0625	0001	4 bit

Average code word length (L):

$$L = \sum_{k=0}^6 p_i \times n_i$$

$$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$$

$$= 2.6250 \text{ bits/message}$$

Entropy of the source (H):

$$H(s_i) = \sum_{k=0}^6 p_i \times \log_2(1/p_i)$$

$$= 0.25 \log_2(1/0.25) + 0.25 \log_2(1/0.25) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) +$$

$$0.125 \log_2(1/0.125) + 0.0625 \log_2(1/0.0625) + 0.0625 \log_2(1/0.0625) = 2.6250 \text{ bits/symbols}$$

$$\text{Code efficiency } \eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$$

$$\therefore \eta = 100\%$$

Note: As the average information per symbol (H) is equal to the average code length (L), the code efficiency is 100%.

**Information Theory and Coding (CM303)**  
**Midterm (40%)**  
**26 Dec 2017**

Instructor: Eng. Hosam Almqadim

Time Allowed: 2 h

Q1. (12 Marks) Consider the following information channel, the channel input A with symbols  $a_1, a_2,$  and  $a_3,$  and probabilities  $p(a_1)=0.6, p(a_2)=0.3,$  and  $p(a_3)=0.1.$  The channel output B with symbols  $b_1, b_2,$  and  $b_3.$  The channel is fully specified by the following channel matrix

$$p(B/A) = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

- a) Draw the schematic of the channel? (2 Marks)
- b) Obtain the output probabilities  $p(b_1), p(b_2)$  and  $p(b_3)$ ? (3 Marks)
- c) Find the Mutual Information  $I(A;B)$ ? (5 Marks)
- d) Find the probabilities and entropy of the 2<sup>nd</sup> order extension Z of A? (2 Marks)

Q2. (10 Marks) In a telegraph source having two independent symbols dot and dash, the dot duration is 0.2 sec and the dash duration is 0.6 sec. The probability of dot occurrence is three times that of dash and the time separation between symbols is 0.1 sec. Find the following:

a) The information rate of this telegraph.  $H(S) \quad R_S$  (5 Marks)

b) The maximum possible information rate with the same average symbol duration. (5 Marks)  
 $\log_2(m) \quad C = \frac{R}{R_S} = \frac{r}{R_S} \quad \log_2(m)$

Q3. (18 Marks) Consider a Discrete Memoryless Source with symbols  $S = \{a,b,c,d,e,f,g\}$  have probabilities  $P(S) = \{2/7, 3/14, 1/7, 1/7, 1/14, 1/14, 1/14\}$  respectively.

- 1. Construct a Huffman code for the source? (4 Marks)
- 2. Compute the average code length of the code. (2 Marks)
- 3. Find the efficiency and the redundancy of the code? (2 Marks)
- 4. Is this code an optimal code? (Justify your answer) (2 Marks)
- 5. Find RI if the DMS generates 1 symbol randomly every 1usec? (2 Marks)
- 6. Encode the following message: aabacaafggefad (2 Marks)
- 7. Draw the decision tree of the code. (2 Marks)
- 8. Decode the following bit stream: 0101000110110010000110111100000001... (2 Marks)

$a \ a \ g \ e \ d \ d \ c \ e \ b \ d \ f \ a$

$H(b/a_1) =$

$H(b/a_i) =$

hmswhmsw

$H(b/a) =$

Good luck



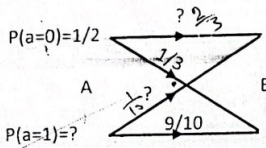
# Information Theory and Coding (CM303)

Midterm Exam (40%)  
16 April, 2016

Instructor: MEng. Hosam Almqadim

Time Allowed: 2h 15min

**Q1. (10 Marks)** Consider the following binary information channel



Determine  $H(A)$ ,  $H(A/B)$ ,  $H(B/A)$  and mutual information  $I(A; B)$  and  $H(P)$

**Q2. (15 Marks)** In a PCM system the voice signal is quantized in 16 levels with the following probabilities:

$P_1 = P_2 = P_3 = P_4 = 0.1$

$P_5 = P_6 = P_7 = P_8 = 0.05$

$P_9 = P_{10} = P_{11} = P_{12} = 0.075$

$P_{13} = P_{14} = P_{15} = P_{16} = 0.025$

a) Calculate the entropy.

b) Calculate the Information rate. Assume  $f_m = 4$  kHz.

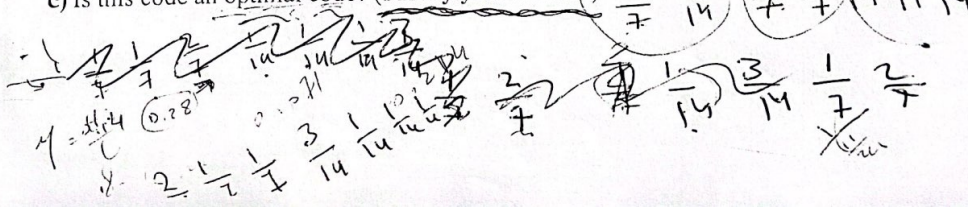
c) Calculate the entropy of the third order extension of this source  $H(z=s^3)$ .

**Q3. (15 Marks)** Consider a Discrete Memoryless Source with symbols  $S_i = 1, 2, 3, 4, 5, 6, 7$  have probabilities  $P(S_i) = \{2/7, 3/14, 1/7, 1/7, 1/14, 1/14, 1/14\}$  respectively.

a) Construct a Huffman code for the source?

b) Find the efficiency and the redundancy of the code?

c) Is this code an optimal code? (Justify your answer)



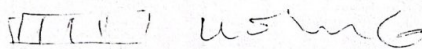


الفصل الدراسي : خريف 2015-2016 اسم الأستاذ/المنسق : حسام الدين الهنشيرى. الزمن : ساعتان.

رقم القيد : .....  
 المجموعة : .....

Q1: In a telegraph source having two independent symbols dot and dash, the dot duration is 0.1s and the dash duration is 0.5s. The probability of dot occurrence is four times that of dash and the time separation between symbols is 0.1s. Find the information rate of this telegraph source? (16 Marks)

Q2) Consider a DMS  $S$  with symbols  $s_i, i=1,2,3,4,5,6$  and with probabilities  $p(s_1)=0.36, p(s_2)=0.24, p(s_3)=0.15, p(s_4)=0.12, p(s_5)=0.08,$  and  $p(s_6)=0.05$ . Construct a Huffman code for the source and find the efficiency of the constructed code? (16 Marks)



Q3) Consider a systematic block code whose parity check equations are:

$$\begin{aligned} b_0 &= m_2 + m_3 + m_4 \\ b_1 &= m_2 + m_3 + m_4 \\ b_2 &= m_1 + m_2 + m_3 \\ b_3 &= m_1 + m_2 + m_3 \end{aligned}$$

Where  $m_i$  are the message bits,  $i=0, 1, 2, 3$ , and  $b_i$  are the check bits,  $i=0, 1, 2, 3$ .

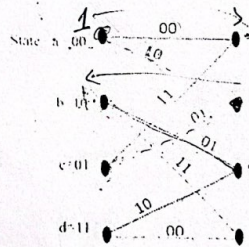
- (a) What are the parameters  $n$  and  $k$ ? Find the generator matrix for the code. (2 Marks)
- (b) What is the minimum Hamming distance? How many errors can the code correct? (2 Marks)
- (c) Is the vector  $[1\ 0\ 1\ 0\ 1\ 0\ 1\ 0]$  a valid codeword? (2 Marks)
- (d) Is the vector  $[0\ 1\ 0\ 1\ 1\ 1\ 0\ 0]$  a valid codeword? (2 Marks)

$$e = d_{min} - 1$$

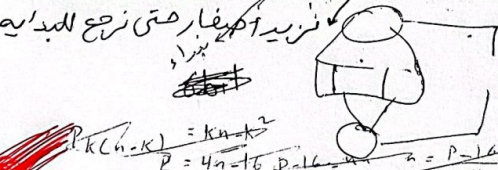
$$t = \lfloor \frac{d_{min}-1}{2} \rfloor$$

Q4: For the given encoder trellis diagram:

- a) Draw the encoder state diagram? (4 Marks)
- b) Draw the encoder? (show your answer) (4 Marks)
- c) Write the generator polynomial of the encoder? (2 Marks)
- d) Encode the message sequence  $11101$  (5 Marks)



عنصرا نقفي متبني للمقابلين وينتبدلوا لارقم نقسم



$$k(n-k) = kn - k^2$$

$$P = 4n - 16, P = 16$$

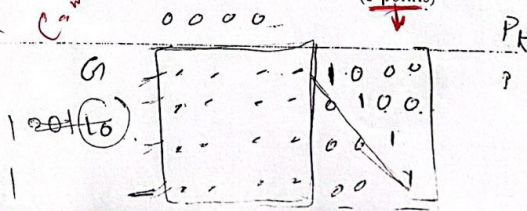
mutual

Given that the mutual information  $I(X, Y) = \sum_{x,y} p(x,y) \log(p(y/x)/p(y))$ , derive the follow  $I(X, Y) = H(X) - H(X/Y)$ ? (5 points)



تمنياتي للجميع بالتوفيق  
 استاذ المادة : م. حسام الدين الهنشيرى

100011001006 | 0111



## Information Theory and Coding (CM303)

Answer of Midterm Exam (40%)

07 June, 2015

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

### Q1. Answer:

Since it BDMS, then the source has two symbols  $s_1$  and  $s_2$ . Let the probability of  $s_1$  is  $p(s_1)=a$  then the probability of  $s_2$  is  $p(s_2)=1-a$ . The entropy  $H(s)$  of this source:

$$H(s) = -a \log_2(a) - (1-a) \log_2(1-a)$$

Note that when  $a=0 \rightarrow H(s)=0$

$$a=1 \rightarrow H(s)=0$$

The maximum entropy can be found by the differentiation of  $H(s)$ :

$$\frac{dH(s)}{da} = \frac{d(-a \log_2(a) - (1-a) \log_2(1-a))}{da} = -[\log_2(a) + \log_2(1-a)]$$
$$\frac{dH(s)}{da} = -\log_2(a) + \log_2(1-a)$$

$$\frac{dH(s)}{da} = \log_2\left(\frac{1-a}{a}\right)$$

The maximum is found when  $\frac{dH(s)}{da} = 0$

$$\log_2\left(\frac{1-a}{a}\right) = 0 \text{ when } \frac{1-a}{a} = 1$$

$$\therefore a=0.5$$

Which means when  $a=0.5$   $H(s)$  is maximum

$$H(s) = -0.5 \log_2(0.5) - (1-0.5) \log_2(1-0.5) = 1 \text{ bit/symbol}$$



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Information Theory and Coding (CM303)

Midterm Exam (40%)  
07 June, 2015

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1. What is the maximum entropy  $H(s)$  for Binary Discrete Memoryless Source (BDMS)? Prove your answer? (10 points)

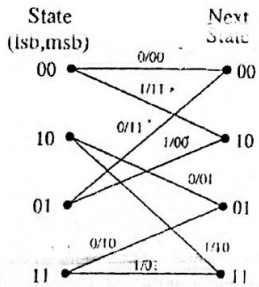
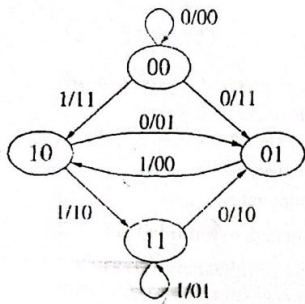
Q2. A telegraph source having two symbols, dot and dash. The dash duration is 0.6 seconds; and the dot duration is two third of the dash duration. The probability of the dot occurring is twice that of the dash, and the time between symbols is 0.2 seconds. Calculate the information rate of the telegraph source? (10 points)

Q3. A discrete memoryless source has an alphabet of seven symbols with probabilities for its output as described in following table:

Symbol	$S_0$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

1. Construct a Shannon-Fano code for the source and calculate the efficiency of coding? (10 points)
2. Construct a Huffman code for the source and calculate the efficiency of coding? And compare the results? (10 points)

Consider the encoder:



$$\mathbf{m} = [1, 1, 0, 0, 1, 0, 1, 0, \dots]$$

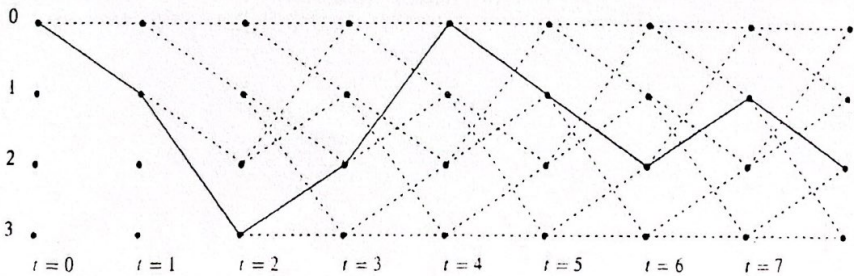
is applied to the encoder, the coded output bit sequence is

$$\mathbf{c} = [11, 10, 10, 11, 11, 01, 00, 01, \dots]$$

The output sequence and corresponding states of the encoder are shown here.

$t$	Input $m_t$	Output $c_t$
0	1	11
1	1	10
2	0	10
3	0	11
4	1	11
5	0	01
6	1	00
7	0	01

Path through trellis corresponding to true sequence



## Q2. Answer:

- Given that:
1. Dash duration: 0.6 sec.
  2. Dot duration:  $2/3 \times 0.6 = 0.4$  sec.
  3.  $P(\text{dot}) = 2 P(\text{dash})$ .
  4. Space between symbols is 0.2 sec.
- Information rate = ?

### 1. Probabilities of dots and dashes:

Let the probability of a dash be "P". Therefore the probability of a dot will be "2P". The total probability of transmitting dots and dashes is equal to 1.

$$\therefore P(\text{dot}) + P(\text{dash}) = 1$$

$$\therefore P + 2P = 1 \quad \therefore P = 1/3$$

$$\therefore \text{Probability of dash} = 1/3$$

$$\text{And Probability of dot} = 2/3$$

### 2. Average information $H(X)$ per symbol:

$$\therefore H(X) = P(\text{dot}) \cdot \log_2 [1/P(\text{dot})] + P(\text{dash}) \cdot \log_2 [1/P(\text{dash})]$$

$$\therefore H(X) = (2/3) \log_2 [3/2] + (1/3) \log_2 [3] = 0.3899 + 0.5283 = 0.9182 \text{ bits/symbol.}$$

### 3. Symbol rate (Number of symbols/sec.):

The total average time per symbol can be calculated as follows:

$$\text{Average symbol time } T_s = [T_{\text{DOT}} \times P(\text{DOT})] + [T_{\text{DASH}} \times P(\text{DASH})] + T_{\text{space}}$$

$$\therefore T_s = [0.4 \times 2/3] + [0.6 \times 1/3] + 0.2 = 0.6667 \text{ sec/symbol.}$$

Hence the average rate of symbol transmission is given by:

$$R_s = 1/T_s = 1.5000 \text{ symbols/sec.}$$

### 4. Information rate ( $R_i$ ):

$$R_i = R_s \times H(s) = 1.5000 \times 0.9182 = 1.72 \text{ bits/sec.}$$

1.3773



**Q3. Answer:**

1. Shannon-Fano code:

Symbols	Probability	Step 1	Step 2	Step 3	Step 4	Code word
S <sub>0</sub>	0.25	0	0			00
S <sub>1</sub>	0.25	0	1			01
S <sub>2</sub>	0.125	1	0	0		100
S <sub>3</sub>	0.125	1	0	1		101
S <sub>4</sub>	0.125	1	1	0		110
S <sub>5</sub>	0.0625	1	1	1	0	1110
S <sub>6</sub>	0.0625	1	1	1	1	1111

Average code word length (L):

$$L = \sum_{k=0}^6 p_i \times n_i$$

$$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$$

$$= 2.6250 \text{ bits/message}$$

Entropy of the source (H):

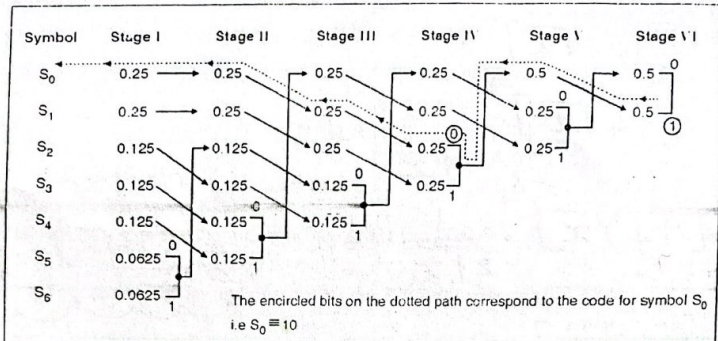
$$H(s_i) = \sum_{k=0}^6 p_i \times \log_2(1/p_i)$$

$$= 0.25 \log_2(1/0.25) + 0.25 \log_2(1/0.25) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.0625 \log_2(1/0.0625) + 0.0625 \log_2(1/0.0625) = 2.6250 \text{ bits/symbols}$$

$$\text{Code efficiency } \eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$$

$$\therefore \eta = 100\%$$

## 2. Huffman Code:



Symbol	Probability	Codeword	Codeword length
$S_0$	0.25	10	2 bit
$S_1$	0.25	11	2 bit
$S_2$	0.125	001	3 bit
$S_3$	0.125	010	3 bit
$S_4$	0.125	011	3 bit
$S_5$	0.0625	0000	4 bit
$S_6$	0.0625	0001	4 bit

**Average code word length (L):**  $L = \sum_{k=0}^6 p_i \times n_i$   
 $= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$   
 $= 2.6250 \text{ bits/message}$

**Entropy of the source (H):**

$$H(s_i) = \sum_{k=0}^6 p_i \times \log_2(1/p_i)$$

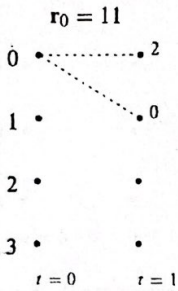
$$= 0.25 \log_2(1/0.25) + 0.25 \log_2(1/0.25) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.125 \log_2(1/0.125) + 0.0625 \log_2(1/0.0625) + 0.0625 \log_2(1/0.0625) = 2.6250 \text{ bits/symbols}$$

**Code efficiency  $\eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$**   
 $\therefore \eta = 100\%$

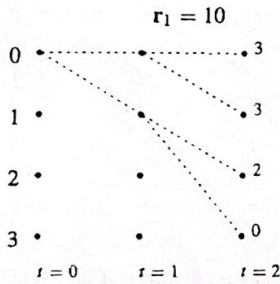
Note: As the average information per symbol (H) is equal to the average code length (L), the code efficiency is 100%.

The algorithm proceeds as follows:

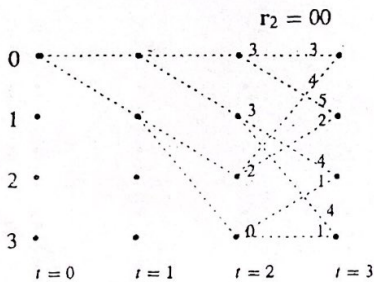
$t = 0$ : The received sequence is  $r_0 = 11$ . We compute the metric to each state at time  $t = 1$  by finding the (Hamming) distance between  $r_0$  and the possible transmitted sequence  $c_0$  along the branches of the first stage of the trellis. Since state 0 was known to be the initial state, we end up with only two paths, with path metrics 2 and 0, as shown here:



$t = 1$ : The received sequence is  $r_1 = 10$ . Again, each path at time  $t = 1$  is simply extended, adding the path metric to each branch metric:

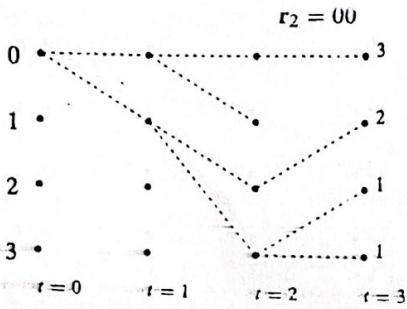


$t = 2$ : The received sequence is  $r_2 = 00$ . Each path at time  $t = 2$  is extended, adding the path metric to each branch metric of each path.

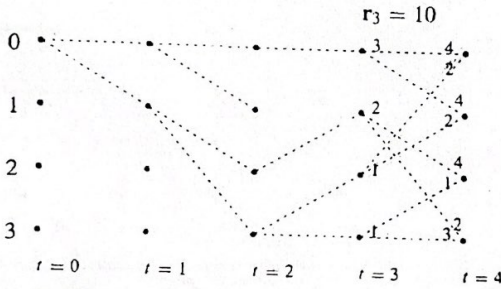




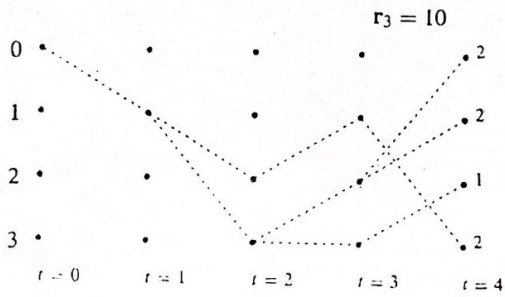
There are now multiple paths to each node at time  $t = 3$ . We select the path to each node with the best metric and eliminate the other paths. This gives the diagram as follows:



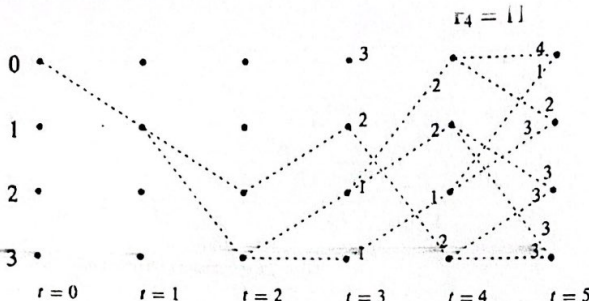
$t = 3$ : The received sequence is  $r_3 = 10$ . Each path at time  $t = 3$  is extended, adding the path metric to each branch metric of each path.



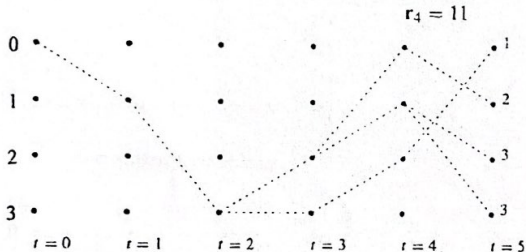
Again, the best path to each state is selected. We note that in selecting the best paths, some of the paths to some states at earlier times have no successors; these orphan paths are deleted now in our portrayal:



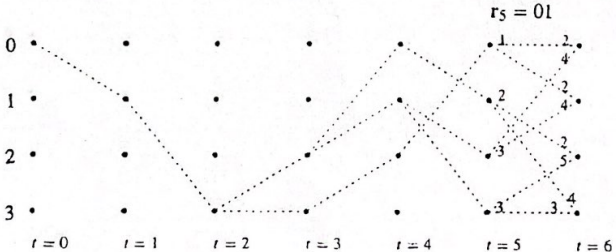
$t = 4$ : The received sequence is  $r_4 = 11$ . Each path at time  $t = 4$  is extended, adding the path metric to each branch metric of each path.



In this case, we note that there are multiple paths into state 3 which both have the same path metric; also there are multiple paths into state 2 with the same path metric. Since one of the paths must be selected, the choice can be made arbitrarily (e.g., at random). After selecting and pruning of orphan paths we obtain:

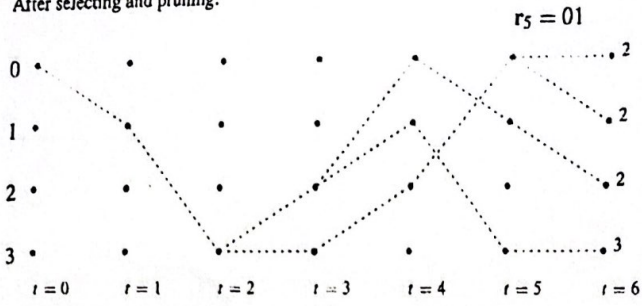


$t = 5$ : The received sequence is  $r_5 = 01$ .

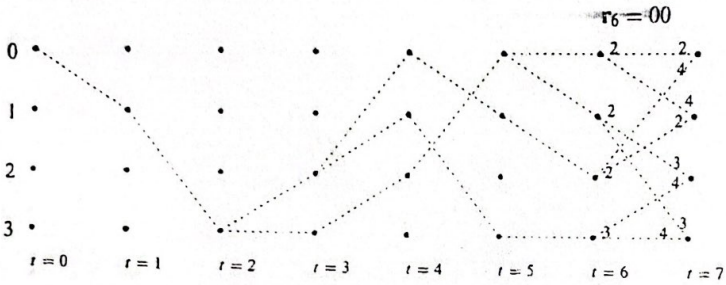


Handwritten notes on the right side of the page, including an arrow pointing upwards and some scribbles.

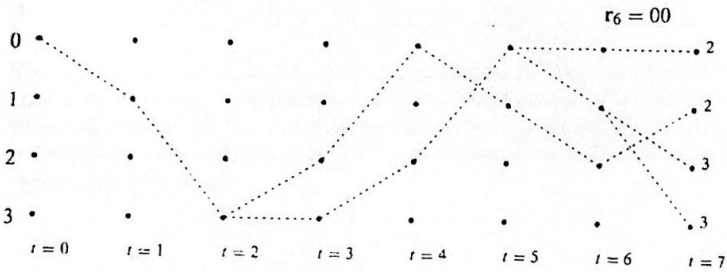
After selecting and pruning:



$t=6$ : The received sequence is  $r_6 = 00$ .

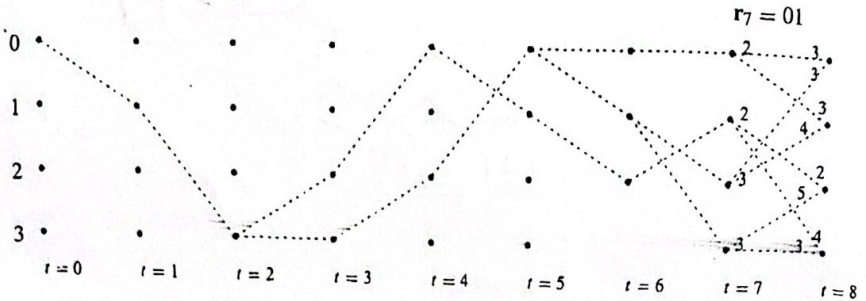


After selecting and pruning:

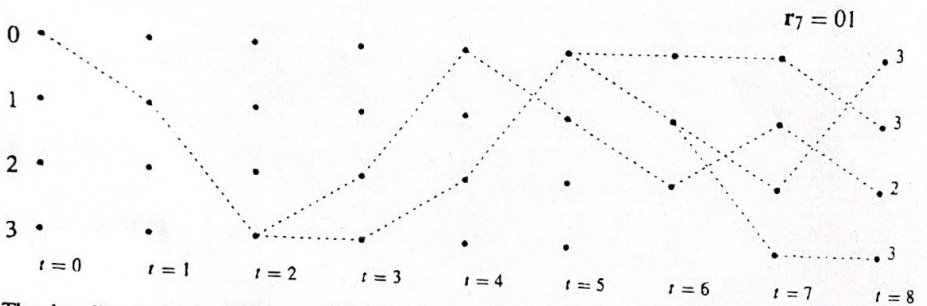




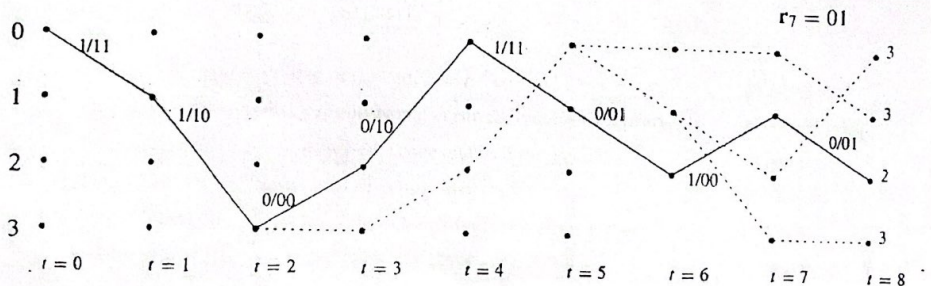
$t = 7$ : The received sequence is  $r_7 = 01$ .



After selecting and pruning:



The decoding is finalized at the end of the transmission (the 16 received data bits) by selecting the state at the last stage having the lowest cost, traversing backward along the path so indicated to the beginning of the trellis, then traversing forward again along the best path, reading the input bits and decoded output bits along the path. This is shown with the solid line below; input/output pairs are indicated on each branch.



# Information Theory and Coding (CM303)

Midterm II (35%)

13 May 2017

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1: (9 Marks) Consider the following information channel, the channel input  $A$  with symbols  $a_1, a_2$ , and  $a_3$ , and probabilities  $p(a_1)=0.6$ ,  $p(a_2)=0.3$ , and  $p(a_3)=0.1$ . The channel output  $B$  with symbols  $b_1, b_2$ , and  $b_3$ . The channel is fully specified by the following channel matrix

$$p(B/A) = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0.5 \end{bmatrix} \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} \begin{matrix} (b_1/a_1) (b_2/a_1) (b_3/a_1) \\ (b_1/a_2) (b_2/a_2) (b_3/a_2) \\ (b_1/a_3) (b_2/a_3) (b_3/a_3) \end{matrix}$$

a) Draw the schematic of the channel?

b) Find the Mutual Information  $I(A;B)$ ?

$$\begin{matrix} p(b_1) \\ p(b_2) \\ p(b_3) \end{matrix}$$

$b/a$

(3 Marks)

(6 Marks)

Q2. (10 Marks) Consider a source with a six-symbol alphabet.  $X_1, X_2, X_3, X_4, X_5$ , and  $X_6$ , with probabilities  $P_1 = 0.20, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02$ , and  $P_6 = 0.04$ , respectively.

A. Find a Shannon-Fano code for this source. *step*

B. Compute the average code length of the code.  $L = \sum P_i \cdot n_i$

C. Design a decision tree of the code?

D. Is the code an optimal code? (justify your answer)

(4 Marks)

(2 Marks)

(2 Marks)

(2 Marks)

Q3 (16 Marks) Given the following code

message	Code Word	message	Code Word
0000	000 0000	1000	1001 1000
0001	110 0001	1001	011 1001
0010	011 0010	1010	110 1010
0011	101 0011	1011	000 1011
0100	110 0100	1100	011 1100
0101	000 0101	1101	101 1101
0110	101 0110	1110	000 1110
0111	011 0111	1111	110 1111

$n = 20$   
 $k = 16$

Aya MOHAMMED

- Find the Generator matrix of the code?  $G [P_{(n-k) \times k} | I_k]$
- Find the error detection capability and error correction capability?
- Find the parity check matrix of the code and its Transpose?
- Is this code a linear code? (Justify your answer)
- Are the generator vectors linearly independent? (Justify your answer)

(4 Marks)

(2 Marks)

(2 Marks)

(2 Marks)

(2 Marks)

Encode the following bit stream 10110010000110111100.....

Draw an encoder of the code?

(2 Marks)

(2 Marks)

$$\begin{matrix} 1011 & \rightarrow & 0001011 \\ 0010 & \rightarrow & 0110010 \end{matrix}$$

Good luck

Good Luck

$u = mG$



Information Theory and Coding (CM303)  
Midterm II (35%)

22 Nov 2016

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1. (5 marks) Given that the mutual information  $I(A;B) = H(A) - H(A|B)$ , derive that  $I(A;B) = I(B;A)$  ?

$H(B) - H(B|A)$

Q2. (10 marks) Consider a DMS Source with symbols  $S_i, i=1,2,3,4$ . Table below lists all possible binary codes

$c_1, c_2, c_3, c_4, c_5, c_6$   
 $c_1, c_2, c_3, c_4, c_5, c_6$   
 $c_1, c_2, c_3, c_4, c_5, c_6$

Table of Codes of the source S						
$S_i$	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

- a) Find which of them distinct codes are? 2, 3, 4, 5, 6
- b) Find which of them prefix-free codes are? 2, 4, 6
- c) Find whether instantaneous codes are existence for these codes? 4, 5
- d) Can you decide which code is the best code for this source, and why?

Q3. (10 marks) Consider a DMS source with symbols  $X_i$ , where  $i=1,2,3$ , and 4, with probability,  $P(X_i) = 0.2, 0.35, 0.05$ , and 0.4, respectively?

- a) Construct a Huffman code for the source?
- b) Find the information rate if the source generates one symbol randomly every  $\frac{1}{15}$  sec?

Q4. (10 marks) Consider the linear block code with the codeword defined by  $U = m_1 + m_2 + m_4, m_1 + m_3 + m_4, m_1 + m_2 + m_3, m_1, m_2, m_3, m_4$

- 1. Find the generator matrix  $G$
- 2. Find all the codewords
- 3. Find the parity-check matrix  $H$
- 4. Find (code bits) message bits, parity bits, code rate, minimum distance, error detection capability and error correction capability.

1	1	1	1	1	0	0	0
1	0	1	1	1	0	0	0
0	1	1	1	0	0	1	0
1	1	0	1	1	0	0	1

$n=8$   
 $k=4$   
 $n-k=4$

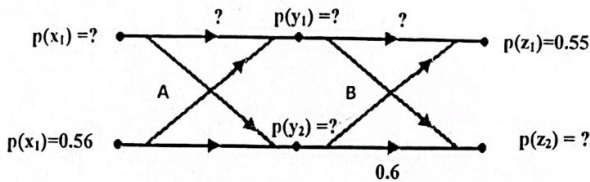
1	1	1	1	1	0	0	0
1	0	1	1	0	1	0	0
0	1	1	1	0	0	1	0
1	1	0	1	0	0	0	1

Good luck





Q1. (12 Marks) For the two Binary Symmetrical Channels (A & B) connected in cascade as shown below, Find the input, output and channels forward probabilities.



Q2. (8 Marks) Consider a DMS Source with symbols  $S_i$ ,  $i=1,2,3,4$ . Table below lists 6 possible binary codes

Table of Codes of the source S						
$S_i$	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

- Find which of them distinct codes are? (2 Marks)
- Find which of them prefix-free codes are? (2 Marks)
- Find weather instantaneous codes are existence for these codes? (2 Marks)
- Can you decide which code is the best code for this source, and why? (2 Marks)

Q3. (24 Marks, 3 each) Consider the Systematic Linear Block Code with the following syndrome look-up table.

Error Pattern (e)	Syndrome (S)
0000000	000
0000001	110
0000010	011
0000100	101
0001000	111
0010000	001
0100000	010
1000000	100

- Find the Generator matrix of the code?
- Find all the Code Words and the Minimum Hamming Distance?
- Find code bits, message bits, parity bits, code rate, the error-detection and error-correction capabilities of the code?
- Write down the Parity Check Equations and draw the Encoder?
- Are the generator vectors linearly independent? (Justify your answer)
- Is this code a linear code? (Justify your answer)
- Encode the bit stream,  $m=110111001001...?$
- If  $r_1 = 1101011$  and  $r_2 = 0101101$  were received, what are the transmitted code-words and original messages? (clearly show the recovery steps)

Q4. (16 Marks) Given a Binary Convolutional Encoder with  $K=3$ , rate  $1/3$ , and Impulse Response 101011010.

- Encode the input sequence bits  $m = 10101?$
- Draw the encoder? (show your answer)
- Draw the Trills diagram of the encoder?

$$\begin{matrix} 100000 \\ 010000 \end{matrix} \begin{bmatrix} 100 \\ 010 \end{bmatrix} \begin{matrix} (5 \text{ Marks}) \\ (6 \text{ Marks}) \\ (5 \text{ Marks}) \end{matrix}$$

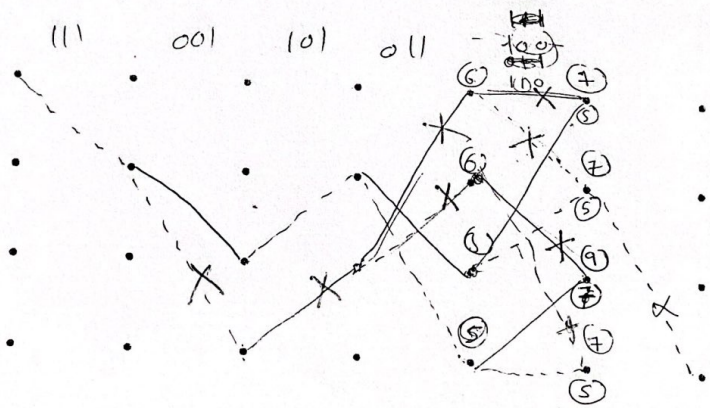
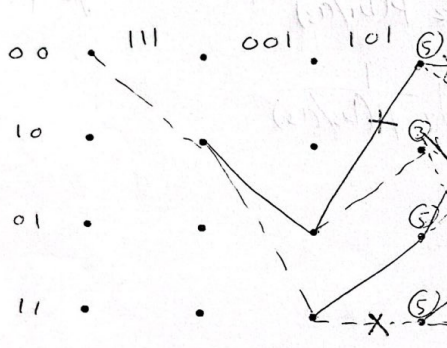
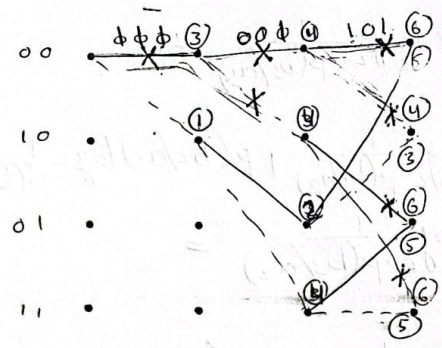
د. ب. ب. ب.

$$G = \begin{bmatrix} P \\ u \end{bmatrix}$$

تمنياي للجميع بالتوفيق  
ا. حسام الدين الهنشيرى

$$I_k H^T = \begin{bmatrix} (I_{n-k}) \\ P \end{bmatrix}$$

$$I_{n-k}$$





قسم الاتصالات. أسئلة الامتحان النهائي لمادة: نظرية المعلومات و الترميز  
 الفصل الدراسي: ربيع 2016 اسم الأستاذ/المنسق: حسام الدين الهنشيوري. رمز المادة: CM 303. الترخيص: 12-06-2016  
 رقم التقيد: ..... الزمن: ساعتان. المجموعة: .....

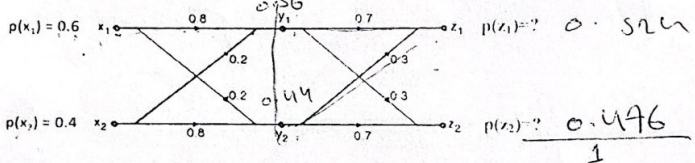
Q1 (16 Marks) In a telegraph source having two independent symbols dot and dash, the dot duration is 0.2 sec and the dash duration is 0.6 sec. The probability of dot occurrence is three times that of dash and the time separation between symbols is 0.1 sec. Find the following:

- A. The information rate of this telegraph. (12 Marks)
- B. The maximum possible information rate with the same average symbol duration. (4 Marks)

Q2 (16 Marks) Consider a source with a six-symbol alphabet  $X_1, X_2, X_3, X_4, X_5,$  and  $X_6$  with probabilities  $P_1 = 0.2, P_2 = 0.01, P_3 = 0.35, P_4 = 0.38, P_5 = 0.02,$  and  $P_6 = 0.04$ , respectively.

- A. Find a Shannon-Fano code for this source. (8 Marks)
- B. Compute the average code length of this Huffman code. (8 Marks)

Q3 (8 Marks) Consider two binary symmetrical channels are connected in cascade as shown below



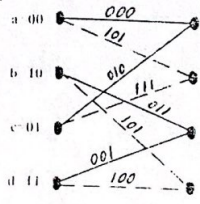
- A. Find the probabilities  $p(z_1)$  and  $p(z_2)$ . (8 Marks)

Q4 (10 Marks) Consider the Systematic Linear Block Code with the following parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- A. If the code is a single error correction code, find the syndrome look-up table. (5 Marks)
- B. For the received vectors  $r_1$  and  $r_2$ , recover the transmitted code-word and the original message  $r_1 = 1101011, r_2 = 0101101$ . (clearly show the recovery steps) (5 Marks)

Q5 (10 Marks) Given the following trellis diagram of a convolutional code.



- A. Decode the received vector  $r$  using Viterbi Decoding.  $R = 101\ 111\ 111\ 100\ 001\ 011$ .
- B. How many errors are there in the received vector  $R$  (identify the error bits) and show the transmitted vector  $T$ .

7 e s

تمنيتي للجميع بالتوفيق  
 استاذ المادة: م. حسام الدين الهنشيوري



# Information Theory and Coding (CM303)

## Midterm II (40%)

14 May 2016

Instructor: MEng. Hosam Almqadim

Time Allowed: 2 hours

Q1: (5 points) Consider a DMS S with symbols  $S_i, i = 1, 2, 3, 4$ . Table below lists 6 possible binary codes.

Table of Codes of the source S						
$S_i$	Code 1	Code 2	Code 3	Code 4	Code 5	Code 6
S1	00	11	0	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

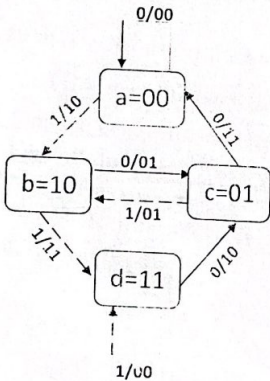
- a) Find which of them **distinct** codes are?  
 b) Find which of them **prefix-free** codes are?

Q2: (20 points) Consider a  $(7,4)$  linear block code with generator matrix G

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- A. Find all the codewords of the code?  
 B. Find the parity-check matrix?  $H$   
 C. [Find code bits, message bits, parity bits] code rate, minimum distance, error detection capability and error correction capability?  
 D. Compute the syndrome for the received vector 1101101. Is this a valid code vector?

Q3: (15 points) For the following state diagram,



Handwritten notes for Q3:  $x^2 + x^4$ ,  $x^2$ ,  $x + x^2$ ,  $x + x^2$

Good luck



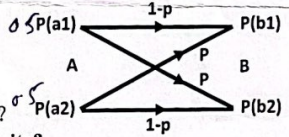
رقم القيد: .....

الاسم: .....

Q1. (10 Marks) In a TV transmission, picture consists of  $2 \times 10^6$  elements, 32 different brightness levels and pictures are repeated at a rate of 32 pictures per second. If the brightness levels have equal likelihood of occurrence and picture elements are independent, find average information rate of this TV source?  
 bit/sym      bit/sec

Q2. (12 Marks, 3 each) For the Binary Symmetrical Channels shown below:

- Find the Channel capacity when  $p = 1$ ,  $p = 0$ ,  $p = 0.5$ , and  $p = 0.3$ ?
- Find the Maximum Capacity of the Channel?
- Find the Input, output, channels forward probabilities,  $H(A)$ , ...  $H(B)$ , and  $H(B/A)$  when the channel works at Maximum Capacity?
- How can you make the channel works at half of its Maximum Capacity?



Q3. (8 Marks, 2 each) Consider a DMS Source with symbols  $S_i, i=1,2,3,4$ . Table below lists 6 possible binary codes

- Find which of them distinct codes are?
- Find which of them prefix-free codes are?
- Find whether instantaneous codes exist for these codes?
- Can you decide which code is the best code for this source, and why?

Table of Codes of the source S						
$S_i$	C1	C2	C3	C4	C5	C6
S1	00	11	01	111	10	0
S2	01	00	1	10	100	1110
S3	01	10	00	110	1000	110
S4	00	01	11	0	1	10

Q4. (15 Marks, 3 each) Consider a Systematic Linear Block Code whose parity check equations are:  
 $P_0 = m_0 + m_1 + m_3$ ,  $P_1 = m_0 + m_2 + m_3$ ,  $P_2 = m_0 + m_1 + m_2$ , and  $P_3 = m_1 + m_2 + m_3$ ,  
 Where  $m_i$  are the message bits,  $i = 0, 1, 2, 3$ , and  $P_i$  are the check bits,  $i = 0, 1, 2, 3$ .

- Find the generator matrix of the code and draw the Encoder?
- Find code bits, message bits, parity bits, code rate, Hamming weight?
- Find Hamming distance, the error-detection and error-correction capabilities of the code?
- Find the syndrome look-up table?
- Are the vectors  $[1\ 0\ 1\ 0\ 1\ 0\ 1\ 0]$  and  $[0\ 1\ 0\ 1\ 1\ 1\ 0\ 0]$  valid codewords? (show the answer steps)

Q5. (4 Marks) Given a Binary Convolutional Encoder with  $K=3$ , rate  $1/3$ , and Impulse Response 101011010. Encode the input sequence bits  $m = 10101$ ? And write down the polynomial equations of the encoder?

Q6. (3 Marks) Consider a  $(4,1,4)$  convolutional encoder with the following generator polynomials:  
 $g_1 = [1010]$ ,  $g_2 = [0101]$ ,  $g_3 = [1110]$ ,  $g_4 = [1001]$ . Draw the encoder and how many states does this encoder have?

Q7. (8 Marks, 2 each) Briefly answer the following:

- How do we measure information content in a message?
- What reduces mutual information between input and output of a channel?
- What is the purpose of source coding and channel coding?
- What are the advantages of convolutional codes over block codes?

1 1 0 1  
1 0 1 1  
1 1 1 0  
0 1 1 1